

A set of similar figures can be drawn for backward-facing waves. Figure 4 *d*) is then unchanged. The others are produced by reflection in the vertical axis, except that appropriate adjustments need to be made depending upon the magnitude and sign of  $u_0$ .

The essential point of the following discussion is that states connected by shocks or by simple waves to an arbitrary state  $(p_0, u_0)$  must lie on fixed curves given by eqs. (32), (34), (35), (36), provided

there are no time-dependent terms in the constitutive relations. This is illustrated in Fig. 5.  $AB$  is represented by eq. (35),  $BC$  by (32),  $BF$  by (34) and  $BD$  by (36).  $AB$  and  $BC$  have a second-order contact at  $B$ , arising from the second order contact between isentrope and Hugoniot at the initial state  $(p_0, V_0)$ . So do  $BD$  and  $BF$ .

Some applications of the foregoing principles are illustrated in Fig. 6-13.

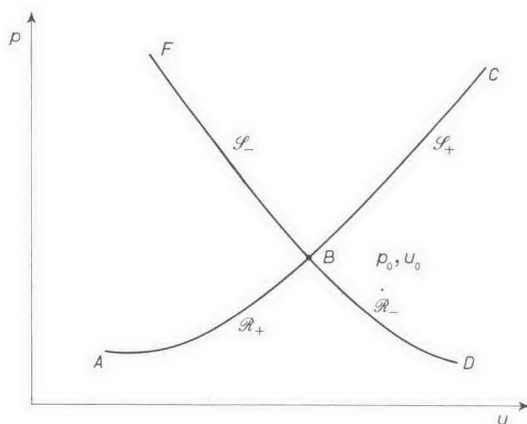


Fig. 5. - Wave transitions in the  $(p, u)$  plane.

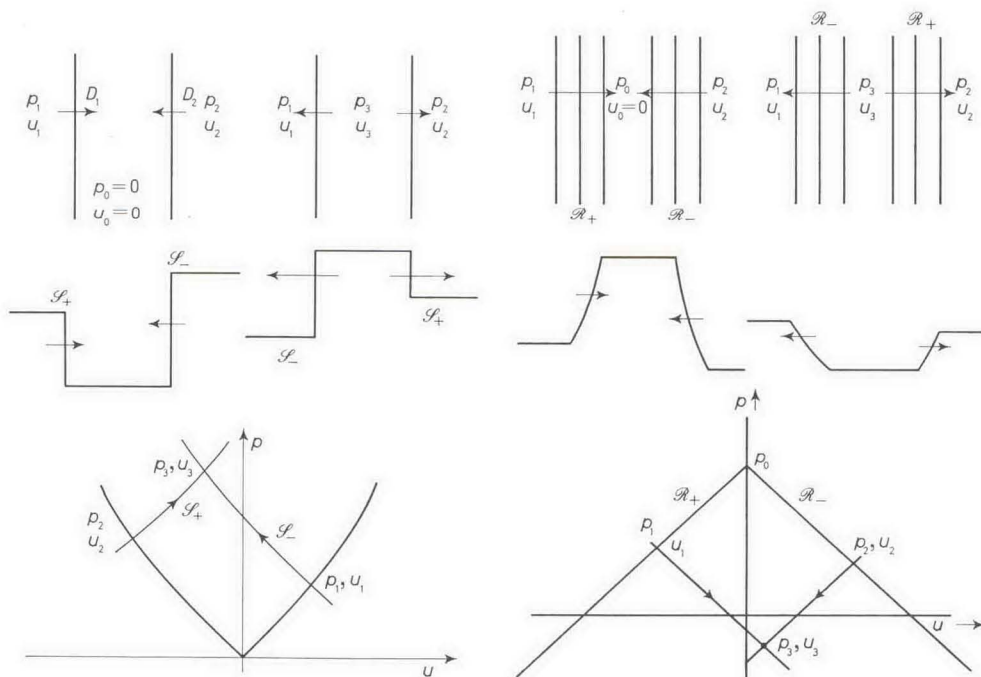


Fig. 6. - Collision of two shocks.

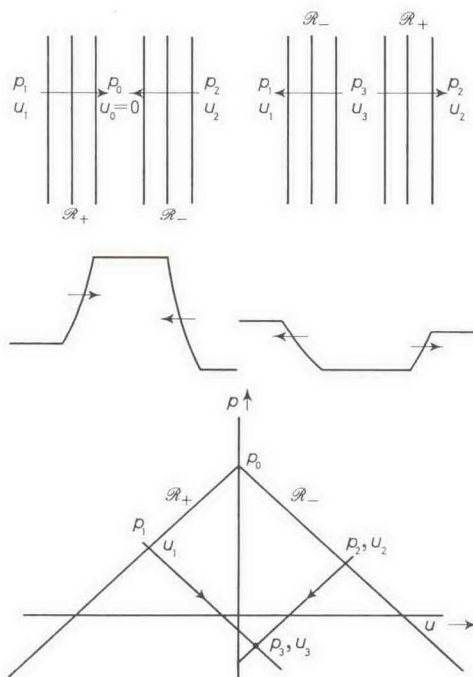


Fig. 7. - Collision of two rarefactions.

Figure 6: Two shocks approach one another, separated by a uniform state  $p_0 = 0 = u_0$ . They are represented by the points  $(p_1, u_1)$  and  $(p_2, u_2)$  in the  $(p, u)$  plane. After collision there are two waves drawing apart, separated by a new state  $(p_3, u_3)$  and running into the uniform states  $(p_1, u_1)$  and  $(p_2, u_2)$ , respectively. The transition from  $(p_2, u_2)$  to  $(p_3, u_3)$  occurs across a forward-facing wave, that from  $(p_1, u_1)$  across a backward-facing wave. The

state between the separating waves must be uniform in  $(p, u)$  because of continuity conditions on  $p$  and  $u$ . All these conditions are satisfied by the intersection of the  $\mathcal{S}_-$  and  $\mathcal{S}_+$  curves from  $(p_1, u_1)$  and  $(p_2, u_2)$ , respectively.

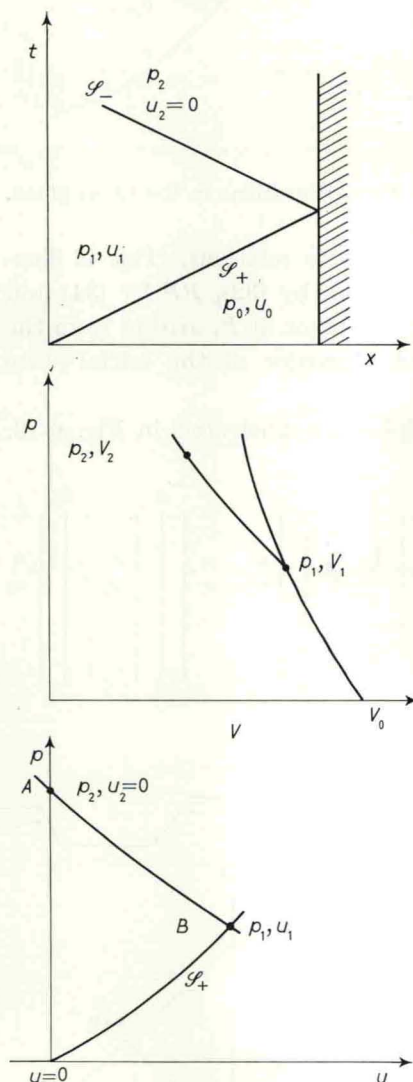


Fig. 8. - Reflection of a uniform shock at a rigid wall.  $p_2/p_1 > 2$  if  $d^2p/dV^2 > 0$ .

Figure 7: Collision of two rarefactions: The argument is analogous to the previous one. The initial state  $p_0, u_0 = 0$ , is connected by rarefactions to  $(p_1, u_1)$  and  $(p_2, u_2)$ . The final state reached from  $(p_1, u_1)$  and  $(p_2, u_2)$  must lie at the intersection of wave transition curves passing through these states and must be a state of uniform  $p$  and  $u$ . This is satisfied by  $(p_3, u_3)$ .

Figure 8: Reflection of a uniform shock at a rigid wall: The initial shock,  $\mathcal{S}_+$ , carries material from  $(u_0 = 0 = p_0)$  to  $(p_1, u_1)$ . After reflection the state  $(p_1, u_1)$  is connected to the final state  $(p_2, u_2 = 0)$  by a backward-facing wave. The final state must lie on the curve for backward-facing transitions in the  $(p, u)$  plane and also on the  $u = 0$  axis. These conditions are satisfied at point  $A$ . Note particularly that if the  $\mathcal{S}_+$  curve is concave upward in the  $(p, u)$  plane,  $p_2 > 2p_1$ . The  $\mathcal{S}_-$  curve through  $B$  is the mirror image of the  $\mathcal{S}_+$  curve in a vertical axis through  $B$ .

Figure 9: Reflection of a uniform shock at a free surface: This is similar to the preceding, but the final state must now lie on the  $p = 0$  axis. For condensed